**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: The work begin after 10 min, so the average time increase from 45min to 55min.

for normal distribution :-

z = (X-μ)/σ

= (60-55)/8

= 0.625

Using this z score in python for probability calculation

P-value = stats.norm.cdf(abs(z\_score))

ie. the probability that the service manager cannot meet his commitment = 0.2676

correct answer is option D.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans: Given that:

µ = 38

σ = 6

Z- score:

Z = (X - µ) / σ

A) More employees at the processing center are older than 44 than between 38 and 44 is False bcz after the calculation we can show that employees between 38 and 44 years of age is more.

B) A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

The above statement is true after the calculation we can show that output is 36.48.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:  From the properties of **normal random variables**,

If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2)  are two independent identically distributed random variables then

* the **sum** of normal random variables is given by

,

* and the **difference** of normal random variables is given by



* When  Z = aX , the **product** of X is given by



Given to find, 2X1

Thus, following the property of multiplication, we get



and following the property of addition,



And the difference between the two is given by



The mean of 2X1 and  X1+X2 is same but the var() of  2X1 is 2 times more than the variance of X1+X2.

The difference between the two says that the two given variables are **identically** and **independently** distributed.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: stats.norm.interval (0.99,100,20)

Using above code in python

Got the two variable a = 48.48 , b = 151.52

So, the option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Mean profits from two different divisions of a company = Mean1 + Mean2

Mean = 5+7

733333333 print('Mean Profit is Rs', Mean\*45,'Million')

from the above python code Mean Profit is Rs 540 Million.

Variance of profits from two different divisions of a company = SD^2 = SD1^2 + SD2^2

SD = (3^2)+(4^2)

print('Standard Deviation is Rs', SD\*45, 'Million')

from the above python code Standard Deviation is Rs 315 Million.

A) print('Range is Rs',(stats.norm.interval(0.95,540,315)),'in Millions')

From the above python code Range is Rs (-77.38865513011706, 1157.388655130117) in Millions.

B) To compute 5th Percentile, we use the formula X=μ + Zσ; wherein from z table,

5 percentile = -1.64

X= 540+(-1.64)\*(315)

print('5th percentile of profit (in Million Rupees) is',np.round(X,2))

from the above python code 5th percentile of profit (in Million Rupees) is 23.4.

C) Probability of Division 1 making a loss P(X<0)

stats.norm.cdf(0,5,3)

from the above python code probability is 0.047.

: Probability of Division 2 making a loss P(X<0)

stats.norm.cdf(0,7,4)

from the above python code probability is 0.040.